Multiple View Geometry

Chapter 4 Estimation - 2D Projective Transformations

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4.2 Different cost functions (7/7)

Sampson error :

- first-order approximation to the point $\hat{X}$
- solvable without iteration

$$\begin{aligned} error function: & \quad \|\delta_{\boldsymbol{X}}\| = \delta_{\boldsymbol{X}}{}^{\top}\delta_{\boldsymbol{X}}. \\ condition: & \quad \mathcal{C}_H(\boldsymbol{X} + \delta_{\boldsymbol{X}}) = \mathcal{C}_H(\boldsymbol{X}) + \frac{\partial \mathcal{C}_H}{\partial \boldsymbol{X}} \delta_{\boldsymbol{X}}. \\ \delta_{\boldsymbol{X}} = (\boldsymbol{X} - \boldsymbol{X}^{\top}) & \quad \mathcal{C}_H(\boldsymbol{X}) = \boldsymbol{A}\boldsymbol{h} \end{aligned}$$

4.2 Different cost functions (7/7)

Sampson error :

The standard way to this solve problem is to use Lagrange multipliers.

$$\begin{cases} f(\boldsymbol{X}) = \|\delta_{\boldsymbol{X}}\| \\ g(\boldsymbol{X}) = \boldsymbol{J}\delta_{\boldsymbol{X}} + \epsilon \end{cases}$$

finally,

$$\|\delta_{\boldsymbol{X}}\|^2 = {\delta_{\boldsymbol{X}}}^2 \delta_{\boldsymbol{X}} = \epsilon^2 (\boldsymbol{J}\boldsymbol{J}^\top)^{-1} \epsilon.$$

4.2 Different cost functions (7/7)

Sampson approximation for a conic : TODO

4.2 Different cost functions (7/7)

Linear cost function : the Sampson error is identical to geometric

error

the variety $\mathcal{V}_H$ defined by $C_H(\boldsymbol{X}) = \mathbf{0}$ is a hyperplane

4.2.7 Another geometric interpretation (7/7)

defferent interpretation in which the set of all measurements is represented by a single point int a measurement space $\mathcal{R}^N$
Now, given a measurement vector $\boldsymbol{x}$ in $\mathcal{R}^N$, the estimation problem is to find the vector $\hat{\boldsymbol{x}}$, closest to $\boldsymbol{x}$, that satisfies the model.

4.2.7 Another geometric interpretation (7/7)

Error in both images :

the set of matched points represents a point in $\mathcal{R}^N$, where $N = 4n$
The vector made up of the coodinates of all the matched points in both images will be denoted $\boldsymbol{x}$.
a feasible choice of points is determinated by a set of $2n + 8$ parameters: the $2n$ coordinates of the points $\hat{\boldsymbol{x}}_i$, plus the 8 independent parameters of the $\boldsymbol{H}$. $\Rightarrow$ sub manifold $S \subset \mathcal{R}^N$ has dimension $2n + 8$ and hence codimension $2n - 8$.
finding the point $\hat{\boldsymbol{x}}$ on S lying closest to x in $\mathcal{R}^N$ is equivalent to minimizing the cost function given by (4.8 reprojection error)
Once $\hat{\boldsymbol{x}}$ is knowns $\boldsymbol{H}$ may be computed.

4.2.7 Another geometric interpretation (7/7)

Error in one images : the measurement space has dimension

$N = 2n$.
finding the closest point on S to the measurement vector $\boldsymbol{X}$ is equivalent to minimizing the cost function (4.6 transfer error).

4.3 Statistical cost functions and Maximum Likelihood estimation(7/7)

In order to obtain a best(optimal) estimate of $\boldsymbol{H}$ it is necessary to have a model for the measurement error(the noise).

we assume that image measurement errors obey a zero-mean istropic Gaussian distribution(no outlier).

we assume that hte noise is Gaussian on each image coordinate with zero mean and uniform standard deviation σ.

PDF:

$$\Pr(\boldsymbol{x}) = \frac{1}{2\pi\sigma^2} e^{-d(\boldsymbol{x},\bar{\boldsymbol{x}})^2/(2\sigma^2)}.$$

4.3 Statistical cost functions and Maximum Likelihood estimation(7/7)

Error in one image :

Then the PDF of the noise-perturbed data is
The *Maximum Likelihood estimate (MLE)* of the homography, $\bar{\boldsymbol{H}}$, maximizes this log-likelihood,

$$\log \Pr(\{\boldsymbol{x}'_i\}|\boldsymbol{H}) = -\frac{1}{2\sigma^2}\sum_i d(\boldsymbol{x}_i, \boldsymbol{H}\bar{\boldsymbol{x}}_i)^2 + \text{constant}.$$

we note that ML estimation is equivalement to minimizing the geometric error functions.

4.3 Statistical cost functions and Maximum Likelihood estimation(7/7)

Error in both images :

$$\log \Pr(\{\boldsymbol{x}_i, \boldsymbol{x}'_i\} | \boldsymbol{H}, \{\bar{\boldsymbol{x}}_i\}) = -\frac{1}{2\sigma^2} \sum_i (d(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)^2 + d(\boldsymbol{x}_i, \boldsymbol{H}\bar{\boldsymbol{x}}_i)^2) + \text{constant}.$$

4.3 Statistical cost functions and Maximum Likelihood estimation(7/7)

Mahalanobis distance :

Miximizing the log-likelihood is then equivalent to minimizing the Mahalanobis distance.

$$\|\boldsymbol{X} - \bar{\boldsymbol{X}}\|_{\Sigma}^{2} = (\boldsymbol{X} - \bar{\boldsymbol{X}})^{\top} \Sigma^{-1} (\boldsymbol{X} - \bar{\boldsymbol{X}}).$$

where Σ and Σ' are the covariance matrices of the measurements in the two images. Finally, if we assume that the errors for all the points $\boldsymbol{x}_i$ and $\boldsymbol{x}'_i$ are independent,

$$\sum \|\boldsymbol{X} - \bar{\boldsymbol{X}}\|_{\Sigma}^{2} + \sum \|\boldsymbol{X}' - \bar{\boldsymbol{X}}'\|_{\Sigma'}^{2}$$

4.4.1 Invariance to image coordinate transformations(7/7)

Does the result of estimation depend on the choice of coordinates in the image?

DLT: at least similarity transformations with scale $s \Rightarrow$ invariance

Exception:

there is no one-to-one correspondence between $\boldsymbol{H}$ and $\bar{\boldsymbol{H}}$ giving rise to the same error ϵ, subject to the constraint $\|\boldsymbol{H}\| = \|\tilde{\boldsymbol{H}}\| = 1$.

the method of transformation leads to a different solution for the computed transformation matrix.

4.4.3 Invariance of geometric error(7/7)

Suppose that euclidean transformation : invariant
minimizing geometric error is invariant to similarity transformations.