# Multiple View Geometry

Chapter 2 Projective Geometry and Transformation of 2D

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2010/4/23

# 2.2.1 Points and lines (1/1)

## Homogeneous representations of lines

*Homogeneous vector*:
An equivalence class of vectors under this equivalence relationship.

$$ax + by + c = 0 \longleftrightarrow (a, b, c)^\top,\ (2a, 2b, 2c)^\top,\ \ldots,\ (ka, kb, kc)^\top,\ \ldots$$

Property:

- The point $\boldsymbol{x}$ lies on the line $\boldsymbol{l}$. $\Leftrightarrow (\boldsymbol{x}, \boldsymbol{l}) = 0$.
- The intersection of two lines $\boldsymbol{l}$ and $\boldsymbol{l}'$ is the point $\boldsymbol{x} = \boldsymbol{l} \times \boldsymbol{l}'$.
- The line through two points $\boldsymbol{x}$ and $\boldsymbol{x}'$ is $\boldsymbol{l} = \boldsymbol{x} \times \boldsymbol{x}'$.

# 2.2.2 Ideal points and the line at infinity (1/2)

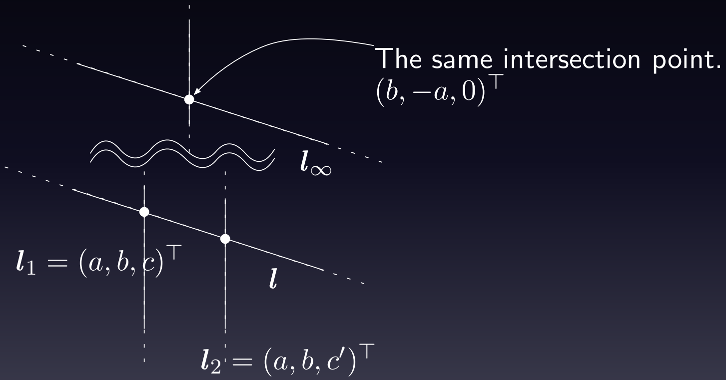
*Ideal points (points at infinity)*:

Homogeneous vectors $\boldsymbol{x} = (x_1, x_2, x_3)^\top$ such that $x_3 = 0$ correspond to infinite points in $\mathcal{R}^2$.

*The line at infinity*:

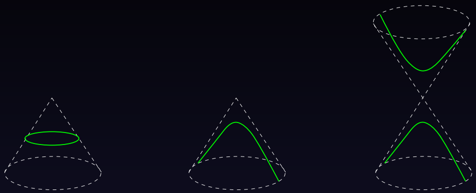
Ideal points lies on a single line $\boldsymbol{l}_\infty = (0, 0, 1)^\top$.

# 2.2.2 Ideal points and the line at infinity (2/2)




- $(b, -a)^\top$ is a *vector tangent to the line*.
- $(a, b)^\top$ is a *line's direction*.

# 2.2.3 Conics and dual conics (1/2)



3 types of conics: ellipse,parabola and hyperbola

## Homogeneous representations of a conic

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Homogenizing $\Rightarrow$

$$(\boldsymbol{x}, \boldsymbol{C}\boldsymbol{x}) = 0, \quad \boldsymbol{C} = \begin{pmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{pmatrix}.$$

# 2.2.3 Conics and dual conics (2/2)

*Tangent lines to conics*:
It's given by $\boldsymbol{l} = \boldsymbol{C}\boldsymbol{x}$.

*Dual conic*:
Dual conic $\boldsymbol{C}^*$ is the adjoint matrix of $\boldsymbol{C}$.

- $(\boldsymbol{x}, \boldsymbol{C}\boldsymbol{x}) = 0$.
- $(\boldsymbol{l}, C^*\boldsymbol{l}) = 0$.

*Degenerate conics*:
If the matrix $\boldsymbol{C}$ is not of full rank, then the conic is termed degenerate.

- Degenerate point conics: $\boldsymbol{C} = \boldsymbol{l}\boldsymbol{m}^\top + \boldsymbol{m}\boldsymbol{l}^\top$
- Degenerate line conics: $\boldsymbol{C}^* = \boldsymbol{x}\boldsymbol{y}^\top + \boldsymbol{x}\boldsymbol{y}^\top$

# 2.3 Projective transformation (1/3)

A planar Projective transformation is a linear transformation on homogeneous 3-vectors represented by a non-singular $3 \times 3$ matrix:

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

or more briefly, $\boldsymbol{x}' = \boldsymbol{H}\boldsymbol{x}$.

Applications:

- Mapping between planes.
- Removing the projective distortion.

# 2.3 Projective transformation (2/3)

## Calculation Homography

$$x' = \frac{x'_1}{x'_2} = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}},$$

$$y' = \frac{x'_2}{x'_2} = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

- These equations are linear in the elements of $\boldsymbol{H}$.
- Four point correspondences lead to eight elements.

# 2.3 Projective transformation (3/3)

Transformation of lines

- $\boldsymbol{l}' = \boldsymbol{H}^{-\top}\boldsymbol{l}$

Transformation of conics

- $\boldsymbol{C}' = \boldsymbol{H}^{-\top}\boldsymbol{C}\boldsymbol{H}^{-1}$.
- $\boldsymbol{C}^{*\prime} = \boldsymbol{H}\boldsymbol{C}^{*}\boldsymbol{H}^{\top}$.