

Multiple View Geometry

Chapter 4 Estimation - 2D Projective Transformations

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4.1 The Direct Linear Transformation (DLT) (1/6)

Minimum solution:

$$\mathbf{x}'_i = \mathbf{H} \mathbf{x}_i$$

The equation can be expressed as follows.

$$\mathbf{x}'_i \times \mathbf{H} \mathbf{x}_i = \mathbf{0}$$

This form will enable a following simple linear solution for \mathbf{H} ,

$$\begin{pmatrix} \mathbf{0}^\top & -w'_i \mathbf{x}_i^\top & y'_i \mathbf{x}_i^\top \\ w'_i \mathbf{x}_i^\top & \mathbf{0}^\top & -x'_i \mathbf{x}_i^\top \\ -y'_i \mathbf{x}_i^\top & x'_i \mathbf{x}_i^\top & \mathbf{0}^\top \end{pmatrix} \begin{pmatrix} h^1 \\ h^2 \\ h^3 \end{pmatrix} = \mathbf{0}$$

where $\mathbf{x}'_i = (x'_i, y'_i, w'_i)^\top$, $\mathbf{H} = (h^1, h^2, h^3)^\top$.

- One point correspondence gives **two** equations.
- Given a set of **four** point correspondence, we can solve for \mathbf{H} .

4.1 The Direct Linear Transformation (DLT) (2/6)

Over-determined solution :

It is given by the following algorithm (DLT).

Objective

Given $n \geq 4$ 2D to 2D point correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$, determine the 2D homography matrix \mathbf{H} such that $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$.

Algorithm

1. For each correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ compute the coefficient matrix. Only the first two rows need be used in general.
2. Assemble the $n \times 9$ matrices into a single $2n \times 9$ matrix \mathbf{A} .
3. Obtain the SVD of \mathbf{A} . The unit singular vector corresponding to the smallest singular value is the solution \mathbf{h} . Specifically, if $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T$ with \mathbf{D} diagonal with positive diagonal entries, arranged in descending order down the diagonal, then \mathbf{h} is the last column of \mathbf{V} .
4. The matrix \mathbf{H} is determined from \mathbf{h} .

4.1 The Direct Linear Transformation (DLT) (3/6)

Inhomogeneous solution :

By imposing a condition $h_j = 1$ for some entry of the vector h , we introduce an alternative Inhomogeneous solution as follows.

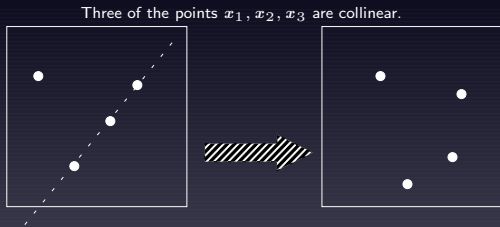
$$\begin{pmatrix} 0 & 0 & 0 & -x_i w'_i & -y_i w'_i & -w_i w'_i & x_i y'_i & y_i y'_i \\ x_i w'_i & y_i w'_i & w_i w'_i & 0 & 0 & 0 & -x_i x'_i & -y_i x'_i \end{pmatrix} h = \begin{pmatrix} -w_i y'_i \\ w_i x'_i \end{pmatrix}$$

However, this method can be expected lead to unstable results in the case where the chosen h_j is close to zero. Consequently, this method is not recommended in general.

4.1 The Direct Linear Transformation (DLT) (4/6)

Degenerate configurations :

Consider a minimal solution,



There can be no projection transform (because a projection transform must preserve collinearity).

4.1 The Direct Linear Transformation (DLT) (5/6)

Degenerate configurations :

In this situation, $\mathbf{H}^* = \mathbf{x}'_4 \mathbf{l}^\top$ is a solution, and..

- \mathbf{H}^* is the unique solution \Rightarrow The solution is a singular matrix.
- There is a further solution $\mathbf{H} \Rightarrow$ 2 dimensional family of transformations $\alpha \mathbf{H}^* + \beta \mathbf{H}$ exists.

The degeneracy problem is not limited to a minimal solution.

- If additional correspondences are supplied which are also collinear, then the degeneracy is not resolved.

4.1 The Direct Linear Transformation (DLT) (6/6)

Solutions from lines and other entities :

It is important how many constraints these entities generate.

entity type	number of constraints
point	2
line	2
conic	5

Mixed entity type:

Correspondences of 2 points and 2 lines can NOT determine homography (because of degenerate).