Multiple View Geometry

Chapter 4 Estimation - 2D Projective Transformations

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4.1 The Direct Linear Transformation (DLT) (1/6)

Minimum solution :

$$\boldsymbol{x}'_i = \boldsymbol{H}\boldsymbol{x}_i$$

The equation can be expressed as follows.

$$\boldsymbol{x}'_i \times \boldsymbol{H}\boldsymbol{x}_i = \boldsymbol{0}$$

This form will enable a following simple linear solution for $\boldsymbol{H}$,

$$\begin{pmatrix} \boldsymbol{0}^\top & -w'_i\boldsymbol{x}_i^\top & y'_i\boldsymbol{x}_i^\top \\ w'_i\boldsymbol{x}_i^\top & \boldsymbol{0}^\top & -x'_i\boldsymbol{x}_i^\top \\ -y'_i\boldsymbol{x}_i^\top & x'_i\boldsymbol{x}_i^\top & \boldsymbol{0}^\top \end{pmatrix} \begin{pmatrix} \boldsymbol{h}^1 \\ \boldsymbol{h}^2 \\ \boldsymbol{h}^3 \end{pmatrix} = \boldsymbol{0}$$

where $\boldsymbol{x}'_i = (x'_i, y'_i, w'_i)^\top$, $\boldsymbol{H} = (\boldsymbol{h}^1, \boldsymbol{h}^2, \boldsymbol{h}^3)^\top$.

- One point correspondence gives two equations.
- Given a set of four point correspondence, we can solve for $\boldsymbol{H}$.

4.1 The Direct Linear Transformation (DLT) (2/6)

Over-determined solution :
It is a given by the following algorithm (DLT).

Objective

Given $n \geq 4$ 2D to 2D point correspondence $\boldsymbol{x}_i \leftrightarrow \boldsymbol{x}'_i$, determine the 2D homography matrix $\boldsymbol{H}$ such that $\boldsymbol{x}'_i = \boldsymbol{H}\boldsymbol{x}_i$.

Algorithm

1. For each correspondence $\boldsymbol{x}_i \leftrightarrow \boldsymbol{x}'_i$ compute the coefficient matrix. Only the first two rows need be used in general.
2. Assemble the $n 2 \times 9$ matrices into a single $2n \times 9$ matrix $\boldsymbol{A}$.
3. Obtain the SVD of $\boldsymbol{A}$. The unit singular vector corresponding to the smallest singular value is the solution $\boldsymbol{h}$. Specifically, if $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^\top$ with $\boldsymbol{D}$ diagonal with positive diagonal entries, arranged in descending order down the diagonal, then $\boldsymbol{h}$ is the last column of $\boldsymbol{V}$.
4. The matrix $\boldsymbol{H}$ is determined from $\boldsymbol{h}$.

4.1 The Direct Linear Transformation (DLT) (3/6)

Inhomogeneous solution :

By imposing a condition $h_j = 1$ for some entry of the vector $\boldsymbol{h}$, we introduce an alternative Inhomogeneous solution as follows.

$$\begin{pmatrix} 0 & 0 & 0 & -x_i w_i' & -y_i w_i' & -w_i w_i' & x_i y_i' & y_i y_i' \\ x_i w_i' & y_i w_i' & w_i w_i' & 0 & 0 & 0 & -x_i x_i' & -y_i x_i' \end{pmatrix} \boldsymbol{h} = \begin{pmatrix} -w_i y_i' \\ w_i x_i' \end{pmatrix}$$

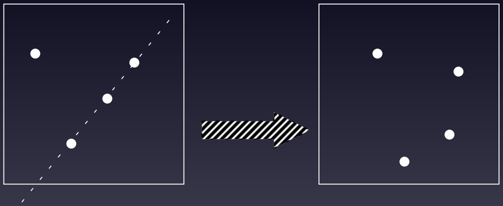
However, this method can be expected lead to unstable results in the case where the chosen h_j is close to zero. Consequently, this method is not recommended in general.

4.1 The Direct Linear Transformation (DLT) (4/6)

Degenerate configurations :

Consider a minimal solution,

Three of the points x_1, x_2, x_3 are collinear.



There can be no projection transform (because a projection transform must preserve collinearity).

4.1 The Direct Linear Transformation (DLT) (5/6)

Degenerate configurations :

In this situation, $\boldsymbol{H}^* = \boldsymbol{x}'_4 \boldsymbol{l}^\top$ is a solution, and..

- $\boldsymbol{H}^*$ is the unique solution $\Rightarrow$ The solution is a singular matrix.
- There is a further solution $\boldsymbol{H}$ $\Rightarrow$ 2 dimensional family of transformations $\alpha \boldsymbol{H}^* + \beta \boldsymbol{H}$ exists.

The degeneracy problem is not limited to a minimal solution.

- If additional correspondences are supplied which are also collinear, then the degeneracy is not resolved.

4.1 The Direct Linear Transformation (DLT) (6/6)

Solutions from lines and other entities :

It is important how many constraints these entities generate.

entity type	number of constraints
point	2
line	2
conic	5

Mixed entity type:
Correspondences of 2 points and 2 lines can NOT determined homography (because of degenerate).