Multiple View Geometry

Chapter 4 Estimation - 2D Projective Transformations

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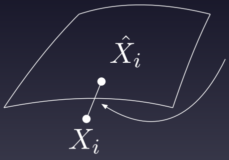
4.2 Different cost functions (5/6)

The image correspondences define an algebraic variety in $\mathcal{R}^4$, and estimating a homography becomes the task of finding a variety.

- A coordinate X_i in $\mathcal{R}^4$ is the pair of image points $(x_i, y_i), ({x'}_i, {y'}_i)$.
- $\boldsymbol{x}'_i \times (\boldsymbol{H}\boldsymbol{x}_i) = \boldsymbol{0}$ presents the intersection X_i of two quadrics.
- A variety $\mathcal{V}_H$ passes (or most nearly passes) through the points X_i.

Perpendicular distance:

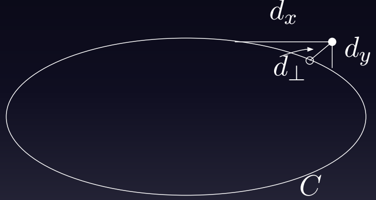


$$
\begin{aligned}
&d_\perp(X_i, \mathcal{V}_H) \\
&\quad = \|X_i - \hat{X}_i\|^2 \\
&\quad = (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 + ({x'}_i - \hat{{x'}}_i)^2 + ({y'}_i - \hat{{y'}}_i)^2 \\
&\quad = \underbrace{d(\boldsymbol{x}, \hat{\boldsymbol{x}}_i)^2 + d(\boldsymbol{x}'_i, \hat{\boldsymbol{x}}'_i)^2}_{\text{reprojection error}}
\end{aligned}
$$

$\hat{X}_i$: the closest point to X_i lying on the variety $\mathcal{V}_H$.

4.2 Different cost functions (6/6)

Conic analogue :



- $d_\perp(X, C)^2 \leq d_x(X, C)^2 + d_y(X, C)^2$
- A conic can be estimated by minimizing $\sum_i d_\perp(x_i, C)^2$.
- Minimizing $\sum_i d_\perp(x_i, C)^2$ cannot be achieved by a linear solution.