Multiple View Geometry

Chapter 4 Estimation - 2D Projective Transformations

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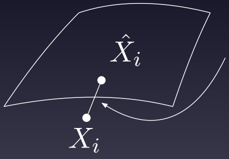
4.2 Different cost functions (5/6)

The image correspondences define an algebraic variety in $\mathcal{R}^4$, and estimating a homography becomes the task of finding a variety.

- A coordinate X_i in $\mathcal{R}^4$ is the pair of image points $(x_i, y_i), ({x'}_i, {y'}_i)$.
- $\boldsymbol{x}'_i \times (\boldsymbol{H}\boldsymbol{x}_i) = \boldsymbol{0}$ presents the intersection X_i of two quadrics.
- A variety $\mathcal{V}_H$ passes (or most nearly passes) through the points X_i.

Perpendicular distance:



$$
\begin{aligned}
&d_\perp(X_i, \mathcal{V}_H) \\
&\quad = \|X_i - \hat{X}_i\|^2 \\
&\quad = (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 + ({x'}_i - \hat{x'}_i)^2 + ({y'}_i - \hat{y'}_i)^2 \\
&\quad = \underbrace{d(\boldsymbol{x}, \hat{\boldsymbol{x}}_i)^2 + d(\boldsymbol{x}'_i, \hat{\boldsymbol{x}}'_i)^2}_{\text{reprojection error}}
\end{aligned}
$$

$\hat{X}_i$: the closest point to X_i lying on the variety $\mathcal{V}_H$.

4.2 Different cost functions (7/7)

Sampson error :

- first-order approximation to the point $\hat{X}$
- solvable without iteration

$$
\begin{aligned}
error function: &\quad \|\delta_{\boldsymbol{X}}\| = \delta_{\boldsymbol{X}}{}^{\top}\delta_{\boldsymbol{X}}. \\
condition: &\quad \mathcal{C}_H(\boldsymbol{X}+\delta_{\boldsymbol{X}}) = \mathcal{C}_H(\boldsymbol{X}) + \frac{\partial \mathcal{C}_H}{\partial \boldsymbol{X}}\delta_{\boldsymbol{X}}. \\
\delta_{\boldsymbol{X}} = (\boldsymbol{X} - \boldsymbol{X}^{\top}) &\quad \mathcal{C}_H(\boldsymbol{X}) = \boldsymbol{A}\boldsymbol{h}
\end{aligned}
$$

The standard way to this solve problem is to use Lagrange multipliers.

$$
\begin{cases}
f(\boldsymbol{X}) = \|\delta_{\boldsymbol{X}}\| \\
g(\boldsymbol{X}) = \boldsymbol{J}\delta_{\boldsymbol{X}} + \epsilon
\end{cases}
$$

finally,

$$
\|\delta_{\boldsymbol{X}}\|^2 = \delta_{\boldsymbol{X}}{}^2\delta_{\boldsymbol{X}} = \epsilon^2(\boldsymbol{J}\boldsymbol{J}^{\top})^{-1}\epsilon.
$$

4.2 Different cost functions (7/7)

Sampson approximation for a conic :