Multiple View Geometry

Chapter 4 Estimation - 2D Projective Transformations

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4.2 Different cost functions (1/4)

Algebraic distance :
DLT minimizes the sum of the next cost function, and we call ϵ_i the *algebraic error vector*.

$$d_{alg}(\boldsymbol{x}_i', \boldsymbol{H}\boldsymbol{x}_i) = \|\epsilon_i\|^2 = \left|\left|\begin{pmatrix} \mathbf{0}^\top & -w_i'\boldsymbol{x}_i^\top & y_i'\boldsymbol{x}_i^\top \\ w_i'\boldsymbol{x}_i^\top & \mathbf{0}^\top & -x_i'\boldsymbol{x}_i^\top \end{pmatrix} \boldsymbol{h}\right|\right|^2$$

Advantage:

- very good accuracy
- a liner (and thus a unique) solution
- computational cheapness

Disadvantage:
Minimize this distance is not geometrically or statistically meaningful.

4.2 Different cost functions (2/4)

Geometric distance :

The estimated homography is the one for which the *transfer error* is minimized.

$$\sum_i d(\boldsymbol{x}_i, \boldsymbol{H}^{-1}\boldsymbol{x}'_i)^2 + \sum_i d(\boldsymbol{x}'_i, \boldsymbol{H}\,\boldsymbol{x}_i)^2$$

assumption : point are measured to a very high accuracy

4.2 Different cost functions (3/4)

Reprojection error :

This method involves estimating a "correction" for each correspondence. Minimizing this cost function involves determining both $\hat{\boldsymbol{H}}$ and a set of subsidiary correspondence $\{\hat{\boldsymbol{x}}_i\}$ and $\{\hat{\boldsymbol{x}}'_i\}$.

$$\sum_i d(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_i)^2 + \sum_i d(\boldsymbol{x}'_i, \boldsymbol{H}\hat{\boldsymbol{x}}'_i)^2$$

subject to $\hat{\boldsymbol{x}}'_i = \hat{\boldsymbol{H}}\hat{\boldsymbol{x}}_i \forall i$.

Notation:

- $\hat{\boldsymbol{x}}$: estimated measured image coordinates values of the points
- $\bar{\boldsymbol{x}}$: true measured image coordinates values of the points

4.2 Different cost functions (4/4)

Comparison of geometric and algebraic distance :

Let $\boldsymbol{x}'_i = (x'_i, y'_i, w'_i)^\top$ and define a vector $(\hat{x}'_i, \hat{y}'_i, \hat{w}'_i)^\top = \hat{\boldsymbol{x}}'_i = \boldsymbol{H}\bar{\boldsymbol{x}}_i$.

Condition:
If $\hat{w}'_i = w'_i = 1$, the two distance are identical. . .

- The two distance are identical.
- The estimated homography $\hat{\boldsymbol{H}}$ is represented by an affine transformation.
- DLT algorithm can minimize gometric distance.